

# On Quasinormal Modes, Black Hole Entropy, and Quantum Geometry

Alejandro Corichi\*

*Instituto de Ciencias Nucleares*

*Universidad Nacional Autónoma de México*

*A. Postal 70-543, México D.F. 04510, MEXICO*

Loop quantum gravity can account for the Bekenstein-Hawking entropy of a black hole provided a free parameter is chosen appropriately. Recently, it was proposed that a new choice of the Immirzi parameter could predict both black hole entropy and the frequencies of quasinormal modes in the large  $n$  limit, but at the price of changing the gauge group of the theory. In this note we use a simple physical argument within loop quantum gravity to arrive at the same value of the parameter. The argument uses strongly the necessity of having fermions satisfying basic symmetry and conservation principles, and therefore supports  $SU(2)$  as the relevant gauge group of the theory.

PACS numbers: 04.60.Pp, 04.70.Dy

## I. INTRODUCTION

Loop quantum gravity (LQG) has become in the past years a serious candidate for a non-perturbative quantum theory of gravity [1]. Its most notorious predictions are the quantization of geometry [2] and the computation of black hole entropy [3, 4]. One of its shortcomings is the existence of a one parameter family of inequivalent quantum theories labelled by the Immirzi parameter  $\gamma$  [5]. The black hole entropy calculation was proposed as a way of fixing the Immirzi parameter  $\gamma$  (and thus the spectrum of the geometric operators) [6]. When a systematic approach to quantum black hole entropy was available [4], this was used to fix the value of  $\gamma$  to be,

$$\gamma = \frac{\ln(2j_{\min} + 1)}{2\pi\sqrt{j_{\min}(j_{\min} + 1)}} \quad (1)$$

where  $j_{\min}$  is the minimum (semi-integer) label for the representations of  $SU(2)$ , responsible for the entropy of the black hole. At that time the most natural assumption was that  $j_{\min} = 1/2$ , giving an Immirzi parameter  $\gamma_{\text{abck}} = \frac{\ln(2)}{\pi\sqrt{3}}$  [4]. Recently, Dreyer made the bold suggestion that there is an independent way of fixing the Immirzi parameter [7], using very little information about LQG. The new approach is based on a conjecture by Hod that the quasinormal mode frequencies  $\omega_{\text{QNM}}$ , for large  $n$  have an asymptotic behavior given by [8],

$$M\omega_{\text{QNM}} = \frac{\ln 3}{8\pi} \quad (2)$$

This conjecture was recently proved analytically by Motl [10]. The conjecture of Hod was within the framework pioneered by Bekenstein in which the area spectrum is assumed to be equally spaced [9]. The argument used by Hod and also by Dreyer goes as follows: One assumes

that the relation between area and mass of a non-rotation black hole is given by  $A = 16\pi M^2$ . Its variations are then  $\Delta A = 32\pi M \Delta M$ . If one assumes that the variation in the mass is due to a quanta radiated with energy  $E_{\text{rad}} = \hbar\omega_{\text{QNM}}$ , and uses the relation (2), one finds that the change in area is given by,

$$\Delta A = 4 \ln(3) l_{\text{P}}^2. \quad (3)$$

Furthermore, Dreyer assumed that the change in area is due to an appearance or disappearance of a puncture with spin  $j_{\min}$ . Thus, one is lead to conclude that the Immirzi parameter is of the form

$$\gamma_{\text{d}} = \frac{\ln(3)}{2\pi\sqrt{j_{\min}(j_{\min} + 1)}} \quad (4)$$

Consistency with the Entropy calculation forces one to take  $j_{\min} = 1$ . Recall that the area contribution from an edge with spin  $j$  is given by

$$A(j) = 8\pi l_{\text{P}}^2 \gamma \sqrt{j(j+1)},$$

and the entropy is given by

$$S = \frac{A}{4l_{\text{P}}^2} \frac{\ln(2j_{\min} + 1)}{2\pi\gamma \sqrt{j_{\min}(j_{\min} + 1)}}$$

As Dreyer recognizes, there are two possible attitudes one might take:

(1) One assumes that the  $j_{\min} = 1$  is due to the minimum possible value that  $j$  can take, even at the kinematical level, in which case one concludes that the gauge group should be replaced by  $SO(3)$  (instead of  $SU(2)$ );

(2) Think of something else.

Giving up the gauge group  $SU(2)$  is, at least to the author, an undesirable step since one would loose the ability of the theory to incorporate fermions. The purpose of this note is to propose a physical argument within loop quantum gravity, that allows to keep  $SU(2)$  as the gauge group, and at the same time have a consistent description with the results of [7]. In fact, our argument

---

\*Electronic address: corichi@nuclecu.unam.mx

is strongly based on the requirement that fermions are contained in LQG. In the following we shall assume that fermions are included and the gauge group is  $SU(2)$ .

This note is organized as follows. In Sec II, we present our argument within the loop quantum gravity formalism. In Sec. III, we reconsider the argument of quasi-normal modes in view of our conclusion of Sec. II, and discuss the implication for the loop quantum gravity program.

## II. LOOP QUANTUM GRAVITY

In this section we shall focus our attention entirely to the LQG formalism. Even when the following considerations are motivated by Dreyer's results, from the logical point of view, they are independent. Let us consider the physical process that would give rise to the QNM frequency [7]: an appearance or disappearance of a puncture with spin  $j_{\min}$ . We can think of this process as being responsible, for instance, for the growth of the BH when an edge that was “free” gets attached to the horizon. The inverse process could occur, say, when the horizon gets excited and then “emits” [13].

Now, in the process of disappearance of the puncture, this edge becomes an open edge in the bulk. If the label of the edge were  $j = 1/2$ , then the only way to make the resulting state gauge invariant is to have a fermion sitting at the end of the open edge. However, this process would violate fermion conservation! One could argue that at the same “time”, another similar process takes place on the horizon such that fermion number is conserved. However, a simpler attitude is to ask for *local* conservation of fermion number. Thus,  $j_{\min} = 1/2$  is forbidden. The minimum allowed value for the “spin” of the resulting free edge is  $j_{\min} = 1$ . In that case, a pair of fermion-antifermion could be attached to the end of the free edge respecting gauge invariance and fermion number conservation. From this perspective, the attachment and dis-attachment to the horizon of edges with  $j = 1/2$  is a “forbidden transition”, and  $j = 1$  is the minimum allowed value of  $j$  that can puncture the black hole horizon, and be responsible for the dynamical process of edge emission and adsorption. Note also that, if this picture is correct, one can only have integer values for  $j$  touching the horizon. Thus, even when  $j = 1/2$  is allowed kinematically, it is the dynamical consistency of the model that restricts the minimum  $j$  involved in the “bulk-surface interaction”.

Given that we are also considering the reverse process, namely when a free edge attaches to the horizon, a process that could be responsible for the black hole acquiring area (and mass). Then one could heuristically expect that most of the area of the black hole comes from this type of process, if the black hole was formed by a dynamical process (as opposed to being an eternal black hole).

A possible conclusion of this argument is that in this

new picture, the main contribution to the entropy comes from  $j_{\min} = 1$ , since these would dominate over the  $j = 1/2$  edges. If that is the case, then one is led to conclude that the value of the Immirzi parameter is the one consistent with this value, namely  $\gamma_d = \frac{\ln(3)}{2\pi\sqrt{2}}$ . Recall that this parameter is a free parameter of the theory. If LQG is going to be physically viable, then one is allowed to make a choice of  $\gamma$  only once. This choice has to be consistent with other situations, or “experiments”. Since the computation of the black hole entropy when matter is included (Maxwell, dilaton, non-minimally coupled scalar field), as well as in more general geometrical scenarios, such as for distorted and rotating horizons [12], also yields *the same value of  $\gamma$* , we still have a consistent theory.

This is the main observation of this note.

*Remark.* Even when a detailed picture (i.e., a Hamiltonian) for the dynamical situation here considered is still to be constructed, the main principles behind our argument, namely gauge invariance and fermion number conservation should be satisfied by *any* such description. In this sense, our argument seems to be robust.

## III. DISCUSSION AND OUTLOOK

In the previous section we have argued that the existence of fermions, together with a simple mechanism for adsorption and emission of edges is consistent with the minimum value of  $j = 1$ , we can now return to the argument by Dreyer. In that case, the change in the area that comes in the thermodynamic argument, complemented with Bohr's correspondence principle, is consistent with the (dis-)appearance of a  $j = 1$  edge.

Needless to say, this is a somewhat heuristic picture, and some issues remain to be solved. For instance:

1. The existence of  $j = 1/2$  edges puncturing the horizon is not forbidden (they would be something like “primordial punctures”), but they must be suppressed. Thus, one needs a dynamical explanation of how exactly the entropy contribution is dominated by the edges with the dynamical allowed value, namely  $j = 1$ .
2. The derivation due to Hod and Dreyer is based on the assumption that one has large black holes for which the entropy is proportional to area and for which the thermodynamical relation between area and mass is valid. One would like to have a complete and systematic understanding of the picture at “small” scales, that is, near the Planck regime.
3. The heuristic physical process of conversion of area quanta to matter quanta via the “emission of an edge” is, of course, very rough. One would like to have a clear picture of this geometry-matter transition.

4. The existence of a universal limit for quasinormal modes frequencies (depending only on the macroscopic parameters of the BH) for non-rotating uncharged black holes is a remarkable fact. An obvious question is whether such a dependence exists for charged and rotating black holes, and if in that case, there is a physical picture within LQG that can produce such frequencies.
5. Closely related to the previous point is the question of whether one should be able to reproduce *all* possible QNM frequencies (such as overtones) from allowed “transitions” in LQG (such as the emission of two edges). This in particular would involve a departure from a “Bohr correspondence principle” to a detailed spectroscopy, which requires a precise understanding of points (3) and (4) above.

The Black Hole entropy calculation is amazing since it combines many nontrivial facts about Chern Simons theory, loop quantum geometry and thermodynamics. If we add the requirement that fermions should be present, we have the highly unexpected result that the (asymptotic) QNM frequencies, that know nothing about  $\hbar$  and quantum mechanics, are given by a simple physical process of

“edge emission”.

In this note we have presented an argument within LQG that supports the minimum value of  $j$  responsible for the BH entropy to be  $j_{\min} = 1$ , and thus making LQG consistent with the QNM frequencies. Indeed, one could even argue for a stronger result. Namely, one could say that a consistent framework for LQG, incorporating fermions and black holes, requires  $j_{\min} = 1$  and in fact, *predicts* the QNM frequencies.

The emerging physical picture is slightly changed from the original considerations of “it from bit” [4] since the fundamental “quanta” that give rise to entropy come from spin 1 contributions, as opposed to spin 1/2. A full understanding of these issues is then a matter of most importance.

### Acknowledgments

I would like to thank A. Ashtekar and J.D. Bekenstein for comments. This work was partially supported by a DGAPA-UNAM grant No. IN112401 and a CONACyT grant No. J32754-E.

- 
- [1] T. Thiemann, “Introduction to modern canonical quantum general relativity,” arXiv:gr-qc/0110034; A. Ashtekar, “Quantum geometry and gravity: Recent advances,” arXiv:gr-qc/0112038.
  - [2] C. Rovelli and L. Smolin, Nucl. Phys. **B442**, 593 (1995) [gr-qc/9411005]; A. Ashtekar and J. Lewandowski, Class Quantum Grav. **14**, A55 (1997) [gr-qc/9602046].
  - [3] K. Krasnov, Phys Rev **D55**, 3505 (1997) [gr-qc/9603025]; C. Rovelli, Phys. Rev. Lett. **77**, 3288 (1996) [gr-qc/9603063].
  - [4] A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, “Quantum geometry and black hole entropy,” Phys. Rev. Lett. **80**, 904 (1998) [gr-qc/9710007]; A. Ashtekar, J. Baez and K. Krasnov, “Quantum geometry of isolated horizons and black hole entropy,” Adv. Theor. Math. Phys. **4**, 1 (2000) [arXiv:gr-qc/0005126].
  - [5] G. Immirzi, Nucl. Phys. Proc. Suppl. **57**, 65 (1997) [gr-qc/9701052].
  - [6] A. Corichi and K. Krasnov, “Ambiguities In Loop Quantization: Area Vs. Electric Charge,” Mod. Phys. Lett. A **13**, 1339 (1998) [arXiv:hep-th/9703177].
  - [7] O. Dreyer, “Quasinormal Modes, the Area Spectrum, and Black Hole Entropy,” arXiv:gr-qc/0211076.
  - [8] S. Hod, “Bohr’s correspondence principle and the area spectrum of quantum black holes,” Phys. Rev. Lett. **81**, 4293 (1998) [arXiv:gr-qc/9812002]; S. Hod, “Gravitation, the quantum, and Bohr’s correspondence principle,” Gen. Rel. Grav. **31**, 1639 (1999) [arXiv:gr-qc/0002002].
  - [9] J.D. Bekenstein, Lett. Nuovo Cimento **11**, 467 (1974); J.D. Bekenstein and V.F. Mukhanov, “Spectroscopy of the quantum black hole”, Phys. Lett. **B360**, 7 (1995); J.D. Bekenstein, “Quantum Black Holes as Atoms”, in *Proceedings of the Eight Marcel Grossmann Meeting on General Relativity*, eds. T. Piran and R. Ruffini (World Scientific, Singapore 1999), pp. 92-111, gr-qc/9710076.
  - [10] L. Motl, “An analytical computation of asymptotic Schwarzschild quasinormal frequencies,” arXiv:gr-qc/0212096.
  - [11] L. Smolin, “Macroscopic deviations from Hawking radiation?”, in *Matters of Gravity* **7** Spring 1996.
  - [12] A. Ashtekar, personal communication.
  - [13] As it has already been argued, this mechanism might not be the one responsible for Hawking radiation, where the frequency spectrum is expected to be different [10, 11].